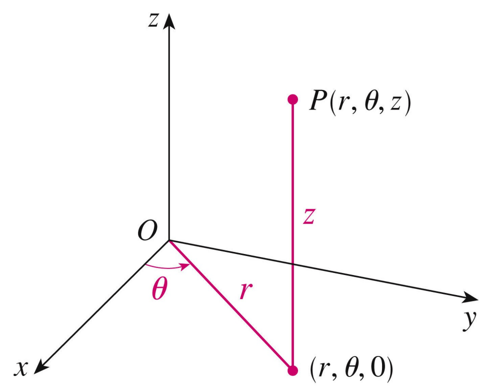
Lesson 28. Triple Integrals in Cylindrical Coordinates

1 Cylindrical coordinates

- Idea: polar coordinates with a z-axis

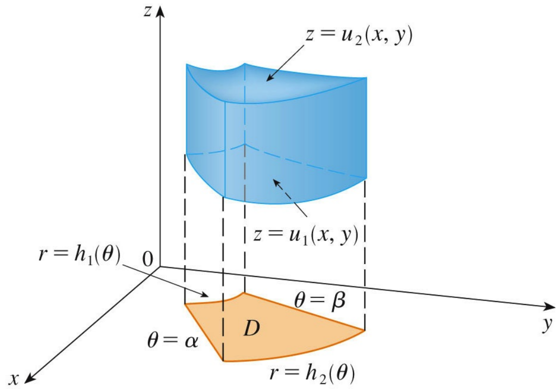


- Converting between cylindrical and rectangular coordinates:

Example 1.

a. Describe the surface whose equation in cylindrical coordinates is $r = 2$.
b. Describe the surface whose equation in cylindrical coordinates is $z = r$.

2 Evaluating triple integrals in cylindrical coordinates



$$\iiint_E f(x,y,z)\,dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\,dz \right] dA$$

$$=$$

Example 2. Set up an iterated integral for $\iiint_E \sqrt{x^2+y^2}\,dV$, where E is the region that lies inside the cylinder $x^2 + y^2 = 16$ and between the planes $z = 2$ and $z = 5$. Use cylindrical coordinates.

Example 3. Convert $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} (x+z)\,dz\,dy\,dx$ to a triple integral in cylindrical coordinates.

Example 4. Set up an iterated integral to find the volume of the solid that is enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 2$. Use cylindrical coordinates.

3 If we have time...

Example 5. Set up an iterated integral to find the volume of the solid above the paraboloid $z = x^2 + y^2$ and below the half-cone $z = \sqrt{x^2 + y^2}$. Use cylindrical coordinates.